Announcements:

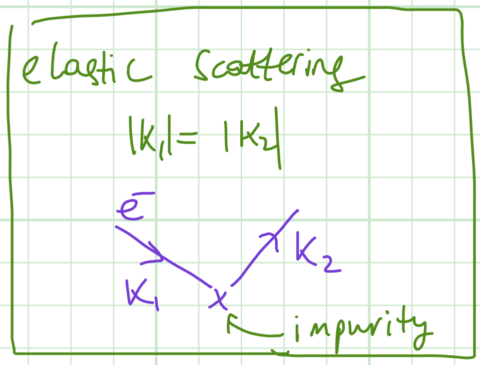
problem 5.22 $\Rightarrow$ replace $1\mu m \to 1cm$ [I think there is a typo in the book]

For plots of the distribution feel free to use eithe Boltzmann or Fermi statistics.

✓

Remark: about Boltzmann equation

Scattering on impurities tends to be elastic



$\Leftarrow$ This is what elastic means, KE before collision = KE after collision

The semi-classical approximation: we assumed that k, x commute. This approximation makes sense in the dilute impurity limit, where

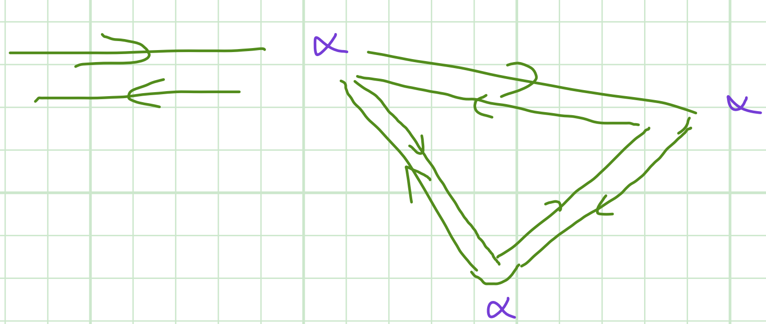
$v_F \tau = \ell \gg \lambda \leftarrow$ typical wave-length

v_F: Fermi-velocity; ℓ: mean free path

D.S.: λ = thermal wavelength $\sim \dfrac{\hbar}{\sqrt{m k_B T}}$ $\Leftarrow$ This makes sense for a Boltzmann gas.

For e^- in metals, we should compare: $\lambda \sim$ Fermi wave length $= \dfrac{1}{k_F}$

The above requirement turns out to be incomplete: at low temperatures it is possible for QM to make a come back due to the interference of semiclassical trajectories

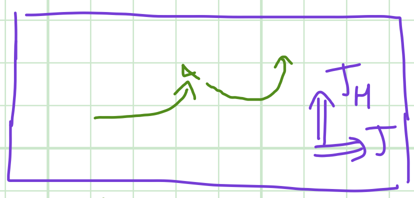


This interference leads to "Anderson localization" $\Rightarrow$ a topic we will revisit.

conjecture: at $T=0$ all materials are either superconductors or insulators [NO METALS]

The Hall effect:

Consider e^- in E + B field



$\vec{E} \rightarrow$ $\otimes \vec{B} = B\hat{z}$

Novelty $\Rightarrow$ Trajectories of e^- will be curved by the B-field.

$\Rightarrow$ Hall current $J_H \perp E$

Since J and E not parallel, we must introduce ρ, σ tensors

$$J_\mu = \sigma_{\mu\nu} E_\nu \quad \text{and} \quad E_\nu = \rho_{\nu\mu} J_\mu \quad \text{with } \rho = \mu^{-1}$$

Question How to compute elements of σ using Boltzmann eqn?

Use note 3 [from last time]:

$$\vec{J}_{drift} = \frac{\vec{J}}{n_0 e} = \frac{e\tau}{m}\vec{E} = \frac{\tau \vec{F}}{m}$$

$\vec{F}$ now has Lorentz force component.

$$F = qE + q v_{drift} \times B \qquad [q = -e, \text{ follow DS notation}]$$

Plugging in the Force into the V_{drift} eqn, we obtain:

$$\Rightarrow V = \frac{\tau}{m}(qE + q\, v \times B) \qquad \text{[Drop the "drift" suffix for clarity]}$$

Explicitly:

$$\begin{cases} V_x = \dfrac{q\tau}{m} E_x + \dfrac{q\tau}{m} B_z V_y \\[2ex] V_y = \dfrac{q\tau}{m} E_y - \dfrac{q\tau}{m} B_z V_x \end{cases}$$

$$V_x = \frac{q\tau}{m} E_x + \left(\frac{q\tau}{m}\right)^2 B_z E_y - \left(\frac{q\tau}{m}\right)^2 B_z^2 V_x \qquad V_x = \frac{\frac{q\tau}{m} E_x + \left(\frac{q\tau}{m}\right)^2 B_z E_y}{1 + \left(\frac{q\tau}{m}\right)^2 B_z^2}$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = qn \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\sigma_{xx} = \frac{qn V_x}{E_x} = \frac{q^2 n \tau}{m} \frac{1}{1+\omega_c^2\tau^2}$$

$$\sigma_{xy} = \frac{qn V_x}{E_y} = \frac{q^2 n\tau}{m} \frac{\left(\frac{q\tau}{m}\right) B_z}{1+\omega_c^2\tau^2} = \frac{q^2 n \tau}{m} \frac{\mu B_z}{1+\omega_c^2\tau^2}$$

What is ω_c ? [Besides $\frac{qB}{m}$?]

$\omega_c = \frac{qB}{m}$ = cyclotrone freq.

$$q\cancel{v}B = \frac{mv^{\cancel{2}}}{r} \qquad \boxed{\frac{v}{r} = \omega = \frac{qB}{m}}$$

Same for V_y we find:

$\sigma_{yy} = \sigma_{xx}$ But $\sigma_{yx} = -\sigma_{xy}$

Why $\sigma_{xy} = -\sigma_{yx}$? Example of Onsager reciprocity relation.

Onsager:

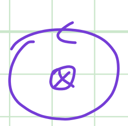
No TR breaking $\sigma_{ij} = \sigma_{ji}$

With TR breaking $\sigma_{ij}(B) = \sigma_{ji}(-B)$

Application to the quantum Hall effect [Integer]

$\Rightarrow$ e^- moving in B-field $\Rightarrow$ semi-classical picture

$\Rightarrow$ Cyclotron orbits



$\updownarrow \; l = \sqrt{\frac{h}{qB}}$

Circumference quantization

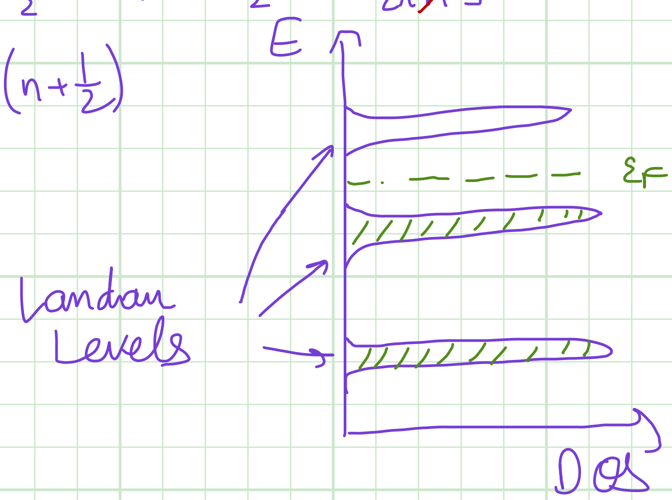
$$2\pi r_c = \frac{nh}{mv} = n \times \lambda_{\text{de Broglie}}$$

$$\frac{mv^2}{2} = \frac{m}{2}(r_c\omega_c)v = \frac{\cancel{m}}{2}\omega_c\left[\frac{nh}{2\pi \cancel{m}}\right] = \hbar\omega_c \frac{n}{2}$$

$\Rightarrow$ quantizing these $E_n = \hbar\omega_c\left(n+\frac{1}{2}\right)$

Density of states in LL?

$\Rightarrow \rho = 1/l^2$



So when $n = \nu/\ell^2$, lowest m-levels are completly filled

$$n = \frac{\nu q B}{h} \Rightarrow B_c = \frac{h n}{q \nu}$$
↖ integer

Conductivity: assume $\tau \to \infty$

⇒ Since LL is completly filled there are no states available for elastic scattering

⇒ scattering must stop!

⇒ plug into formulas for conductivity in B-field

$$\sigma_{xx} \to \frac{q^2 \tau n/m}{1 + \omega_c^2 \tau^2} \sim \frac{1}{\tau} \to 0$$

$$\sigma_{xy} \to \frac{q^2 \cancel{\tau} n/m}{\cancel{1} + (\omega_c \cancel{\tau})^2} \quad \frac{q \cancel{\tau}}{m} B = \frac{q n}{B} = \nu \frac{q^2}{h}$$ ⇐ result you have seen before

⇒ what about resistivity?

$\rho \Rightarrow$ note $\rho_{xx} = \sigma_{xx} = 0$!

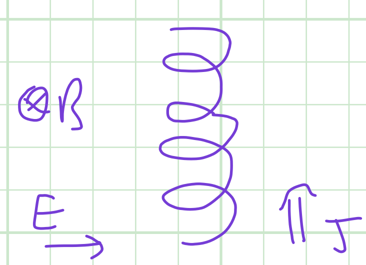
$$\rho = \begin{pmatrix} 0 & -1/\sigma \\ 1/\sigma & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

⇒ why is σ_{xy} non-zero? [This explanation is is a bit too classical]

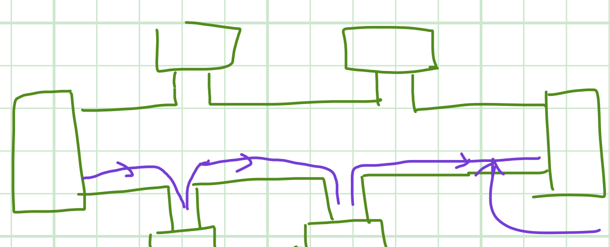
The really hand wavig

"spiralling orbits" + no scattering

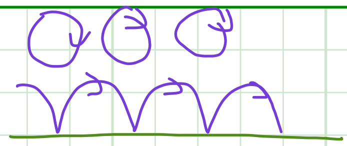
⇒ e^- flow ⊥ to E field



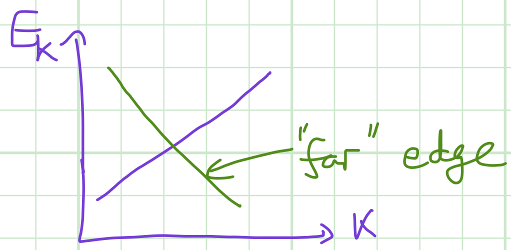
⇒ why σ_{xy} is quantized?



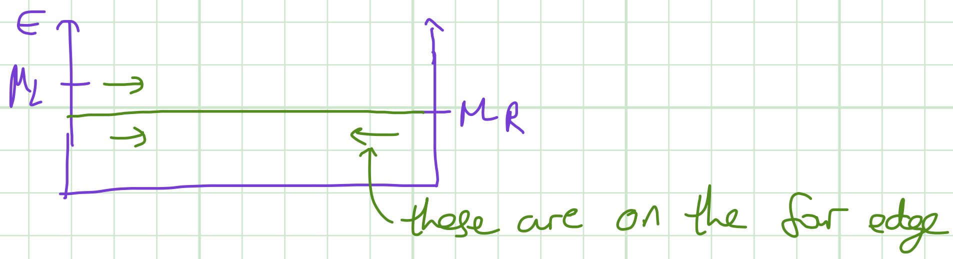
"skipping" orbits ⇒ chiral ⇒ no [back] scatter



$\bar{e}$ only go one way next to the edge!



Conductivity of a quantum channel:



$$J = e\int_{\mu_R}^{\mu_L} \underbrace{\frac{1}{h}\frac{\partial E_k}{\partial k}}_{\Uparrow\ \text{velocity}} dk = \frac{e^2 V}{h} \qquad \sigma = \frac{J}{V} = e^2/h \Leftarrow \text{per channel!}$$

$$L\frac{\cancel{ML^2}}{T^{\cancel{2}}}\,\frac{\cancel{T}}{\cancel{ML^2}} \quad (\checkmark)$$

$$\mu_L - \mu_R = eV$$

⇒ Now we have to figure out where the voltage drops are!

⇒ come back to this subject soon(ish)

⇒ Defects in lattices:

(1) Point defects: